A Theory of Eurobonds

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Abstract

We provide a structural model of sovereign credit risk, where the risk premium paid by the government is linked to some key economic variables of a country: public debt and deficit, GDP growth. This model is then applied to measure the impact of splitting the public debt into a senior and a junior tranches and the effect of introducing Eurobonds: in the latter case, tranching is coupled with a cross-guarantee among eurozone countries and with a cash collateral. We show both in theory and in numerical estimates that eurobonds are able to lower the overall cost of servicing the public debt for some (high debt) countries in the euro area, without increasing the cost for the other ones. Moreover, they are likely to give governments an incentive to curb their deficits, due to the higher marginal cost of debt.

Keywords. Eurobonds, Sovereign debt, Credit risk, Interest rates.

JEL Codes. H63, G12.

1 Introduction

In the policy debate over the sovereign debt crisis in Europe, a widely discussed proposal is that part of the public debt in the euro area should be converted into "eurobonds".[1] More precisely, a European Debt Agency (EDA) should issue bonds in the open market, and lend money to the public sectors of the euro area, up to a threshold level of the debt-to-GDP ratio. Additional features of the proposal are: (i) the claim of the EDA should be senior relative to the other public debt securities; (ii) the creditworthiness of the Eurobonds issued by the EDA should be enhanced by the joint guarantee of the governments of the euro area.

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[1]The proposal has been supported, among others, by Monti (2010) and Juncker -Tremonti (2010).

We believe that the proposal has some merits, that we detail in this paper. In a nutshell, it should be able to lower the cost of servicing the public debt for some countries in the euro area, without increasing the cost for the other ones. Moreover, it is likely to give governments the incentive to curb their deficits. Contrary to other contributions in this area[2], we support our arguments through both a formal theoretical model and some numerical considerations providing a quantitative assessment of the likely impact of introducing Eurobonds into the Euro area.

Our starting point is a structural model of sovereign credit risk, where the default probability of a public obligor, and consequently the interest rate spread paid over the market risk free rate, depend on: i) the expected path of the surplus/deficit of the government; ii) the initial level of debt-to-GDP ratio and iii) expected GDP growth. This model is then applied to the case where the public debt of a country is split between a senior and a junior tranche. This tranching operation at the national level (*domestic tranching*) does not change the overall cost of debt: the gain for the public obligor on the senior tranche is offset by the additional cost on the junior tranche. This property is consistent with previous theories of debt, both corporate and public, stating that the Modigliani - Miller theorem holds for a public obligor, just like for a private corporation: a pure change in the structure of its obligations is irrelevant, unless there is some additional effect due to asymmetric information or to incentives to change the fiscal policy.[3]

The main contribution of our paper is the application of this framework to the case of the Eurobonds proposal. The crucial features are the following:

- threshold limit: governments can borrow from the EDA up to a specific level of the debt-to-GDP ratio (say 40%);
- seniority: the claim of the EDA is senior relative to the other governments' liabilities;
- cross-guarantee: the Eurobonds, issued by the EDA, are guaranteed by the primary surplus of the euro area as a whole;
- diversification: thanks to the cross-guarantee, the worsening of the public sector balance in some countries can be offset by an opposite evolution in some other counties of the euro area;

In our specific version of the Eurobond structure we include two additional features:

- cash collateral: the EDA is endowed with a cash deposit, funded by governments, equal to the expected loss on its own overall exposure with Euro area governments. This is made to ensure that Eurobonds can be issued

[2]See for example Delpla - Weizsacker (2010).
[3]See Canziani, Giavazzi, Manasse and Tabellini (1994).

at the risk free rate, providing a solution to the argument against them raised by AAA countries (mainly Germany).[4]

- To avoid cross-country subsidization, we propose that the burden of the cash deposit should be shared among countries in proportion to their riskiness: as an actuarial measure of this burden we propose the cash deposit that each of them should post if they had to issue senior debt of their own in a *domestic* tranching scheme in order to pay the risk free rate.

Our setting, based on the comparison of *domestic tranching* and Eurobonds, enables us to evaluate benefits and costs of the Eurobond proposal. The effect of lowering the average cost of public debt is obtained by means of the cross-guarantee and the diversification effect. Our model provides a straightforward technical way to measure these two effects. Then, the cash collateral provision enables the EDA to pay the riskless interest rate on the Eurobonds issued and to pass on this benefit to the sovereign borrowers, who in turn can borrow from the EDA at the riskless rate.

Note that with this structure, even those countries enjoying the highest credit standard (e.g. Germany), such that they would be able to issue senior debt at the riskless rate without posting any collateral under domestic tranching, would not bear any additional cost from participating in the Eurobond scheme, since they would not be asked to contribute to the cash deposit (or they would be required to do so for a very small share). The actuarial fairness of this stems from the fact that countries that provide high quality guarantee to the EDA should be required to post less cash guarantee. In addition, Eurobonds would not be a source of additional interest burden for these countries, since by definition they could not pay less than the riskless rate and the Eurobonds would be designed to pay the same rate.

Finally, the cost of junior debt would be the same with domestic tranching and with Eurobonds. In a scenario of invariant fiscal policy, the increase of the interest rate paid on national junior debt relative to the current cost could be quite substantial. This increase should provide an incentive to limit the issues of national bonds, which are going to be very costly. Therefore, the cross-guarantee would not come at the cost of any moral hazard effect, since the marginal cost of debt would go up − while the average cost would go down. Of course, for the incentive effect to work it is crucial that Eurobonds cover only a fraction of government debt, at least for high debt countries.

The plan of the paper is as follows. In Section 2 we describe a structural model of sovereign debt risk. In Section 3 the model is linked to credit spreads (namely asset swap spreads) that represent the market measure of the probability of

[4] A cash reserve is a credit enhancement currently used by the EFSF, together with the 120% overguarantee provided by the eurozone governments. Following the decisions taken in June 2011, the only credit enhancement used by the EFSF starting from 2012 should be a 165% overguarantee. Note that these credit enhancements imply a strong wealth transfer from safe to risky countries, since their burden is allocated in proportion to each country share in the ECB capital, independently of its creditworthiness. The same holds for the subscribed capital of the ESM, which will become operational in 2013.

default of the public obligor. In Section 4 the model is extended to allow for debt tranches of different seniority. In Section 5 the model is applied to derive the credit risk of a Eurobond, that is a debt issue jointly guaranteed by the countries of the Euro area. In Section 6 we present a numerical proposal of a Eurobond structure. In Section 7 we draw the main policy considerations that arise from the model and the numerical example. Section 8 will conclude.

2 A Structural Model of Sovereign Credit Risk

Assume a public obligor issuing the amount of long term debt D_0 at time t_0. Define a set of dates $\{t_1, t_2, \ldots, t_n\}$ in which the obligor will pay interest and principal. For the sake of simplicity assume that payments are made in a single instance every year ($t_i - t_{i-1} = 1, \forall i$). Furthermore, we assume that:

- debt D_0 is issued in the form of coupon bond with maturity t_n, paying annual coupons: debt is issued at par and principal is repaid in one instance at time t_n (*bullet bond* structure), so that at time t_0 the market value of debt equal to its face value;
- each year new debt is issued or excess cash is invested at the same rate of return (buying back existing debt), so that debt may increase or decrease depending on the difference between interest rate payments and primary surplus (that is the budget surplus before interest);
- default is assumed to possibly occur only at the final date t_n when the Government tries to roll-over the debt accrued from time t_0.

Formally, assume a given trajectory of the primary surplus $\{S_1, S_2, \ldots, S_n\}$. Denote by

$$v(t_{i-1}, t_i) = \frac{1}{1 + r(t_{i-1}, t_i)}$$

the discount factor giving the present value at time t_{i-1} of 1 Euro due from the public obligor at time $t_i(r(t_{i-1}, t_i)$ is the corresponding interest rate). In each year t_i the value of debt is

$$D_i = D_{i-1} + \left(\frac{1}{v(t_{i-1}, t_i)} - 1 \right) D_{i-1} - S_i = \frac{D_{i-1}}{v(t_{i-1}, t_i)} - S_i$$

where S_i is the primary surplus. Substituting D_{i-1} into D_i, we have

$$D_i = \frac{D_{i-2}}{v(t_{i-1}, t_i)v(t_{i-2}, t_{i-1})} - \frac{1}{v(t_{i-2}, t_{i-1})} S_{i-1} - S_i$$

Using the property of the discount factor

$$v(t_{i-2}, t_i) = v(t_{i-2}, t_{i-1})v(t_{i-1}, t_i)$$

we can rewrite

$$D_i = \frac{D_{i-2}}{v(t_{i-2}, t_i)} - \frac{v(t_{i-1}, t_i)}{v(t_{i-2}, t_i)} S_{i-1} - S_i$$

Working out the recursion back to the origin t_0, we finally compute

$$D_n = \frac{D_0}{v(t_0, t_n)} - \frac{1}{v(t_0, t_n)} \sum_{i=1}^{n} v(t_0, t_i) S_i \tag{1}$$

for each trajectory of the primary surplus S. We now define the default event. We assume that there is a threshold value beyond which at time t_n the debt will not be rolled over, bringing about a default event. So, there exists a value D_K such that default occurs if $D_n > D_K$. In most cases this threshold can be thought of as a market confidence limit, that should be modeled accordingly. We may also think of D_K as a constitutional limit, self-imposed by a country or by a regulatory body, to the amount of debt that its government is allowed to issue: this is actually the case of the US and, as we will see, of the Eurobond proposal. Then, the probability of a default event taking place at time t_n is

$$\mathbb{P}(D_n > D_K) = \mathbb{P}\left(\frac{D_0}{v(t_0, t_n)} - \frac{1}{v(t_0, t_n)} \sum_{i=1}^{n} v(t_0, t_i) S_i - D_K > 0 \right)$$

If we now add and subtract D_0 and multiply the inequality times $v(t_0, t_n)$

$$\mathbb{P}(D_n > D_K) = \mathbb{P}\left(D_0(1 - v(t_0, t_n)) - \sum_{i=1}^{n} v(t_0, t_i) S_i + v(t_0, t_n)(D_0 - D_K) > 0 \right)$$

We now remind the standard definition of the par yield R_n for maturity t_n. We have

$$R_n = \frac{1 - v(t_0, t_n)}{\sum_{i=1}^{n} v(t_0, t_i)} = \sum_{i=1}^{n} \omega_i f(t_0, t_{i-1}, t_i)$$

where $f(t_0, t_{i-1}, t_i)$ is the forward rate at time t_0 for an investment starting at time t_{i-1} and obtaining 1 Euro at time t_i from the public obligor and

$$\omega_i = \frac{v(t_0, t_i)}{\sum_{i=1}^{n} v(t_0, t_i)}$$

are weights. The par yield (or swap rate) is a weighted average of the forward rates. The default probability can then be rewritten as

$$\mathbb{P}(D_n > D_K) = \mathbb{P}\left(D_0 R_n - \sum_{i=1}^{n} \omega_i S_i + \omega_n (D_0 - D_K) > 0 \right)$$

Now assume a very simple model for S_i. In order to allow for non-stationarity, we model the ratio of primary surplus to GDP, defined as $s_i = S_i / Y_i$, where Y_i is GDP at current prices. We assume the dynamics

$$s_i = \bar{s} + \epsilon_i$$

where $\epsilon_i \sim \mathbb{N}(0, \sigma^2)$ are assumed to be iid and normally distributed disturbances. Now, assume the GDP is growing at rate g (that we assume deterministic) and define

$$\eta_i = \omega_i (1+g)^i$$

Then,

$$\sum_{i=1}^{n} \omega S_i = \bar{s} \sum_{i=1}^{n} \eta_i + \sum_{i=1}^{n} \eta_i \epsilon_i$$

By the normality assumption of the innovations ϵ_i we then have:

$$\sum_{i=1}^{n} \eta_i \epsilon_i \sim \mathbb{N}\left(0, \sigma^2 \sum_{i=1}^{n} \eta_i^2\right)$$

We can now define a concept of *distance to default* (DD) for public debt similar to that used for corporate borrowers. Setting $d_i = D_i / Y_i$ we have in fact that

$$DD(t_0, t_n) = \frac{\bar{s} \sum_{i=1}^{n} \eta_i - d_0 R_n + \omega_n (d_K (1+g)^n - d_0)}{\sigma \sqrt{\sum_{i=1}^{n} \eta_i^2}}$$

Then, the probability of default at time t_n is given by

$$\mathbb{P}(D_n > D_K) = \Phi\left(-\frac{\bar{s} \sum_{i=1}^{n} \eta_i - d_0 R_n + \omega_n (d_K (1+g)^n - d_0)}{\sigma \sqrt{\sum_{i=1}^{n} \eta_i^2}}\right)$$

where $\Phi(z)$ denotes the standard normal distribution. Default probability is then higher the lower the difference between: i) the average primary surplus and the par-yield rate paid on the bond; ii) the default threshold value of debt and current debt value.

If we now add a *Loss Given Default* (LGD) figure, the expected loss EL_n on investment horizon t_n is defined as

$$EL_n = \mathbb{P}(D_n > D_K) LGD$$

Remark 2.1. *An important feature of the default probability is that the sign of the relationship between level of debt and probability of default depends on the value of the par yield and the rate of growth of GDP. To be clear, assume a setting in which debt has to be stabilized at level d_0, so that $d_k = d_0$. It is easy to check that in this case, if $R_n = g$, the probability of default is independent of the level of debt d_0 and only depends on the expected primary surplus path. If $R_n > g$, then the probability of default increases with the amount of debt. The opposite occurs if $R_n < g$. The intuition behind this is quite the same as the rule of debt sustainability. If the rate of growth of GDP is higher than the par yield, it means that the cash amount of debt that it would be possible to roll over at maturity will be higher, implying that the probability of default would be lower.*

3 Credit Spreads

We now connect the expected loss figure to the credit spread observed in the market, with respect to a risk free rate term structure. We implicitly assume that this term structure is still available, even though this is highly questionable. However, the same analysis would be carried out with respect to every benchmark curve that could be suitably chosen as the closest to default free. Denote by $v^*(t_0, t_i)$ the benchmark discount factor curve, and $sr(t_0, t_n)$ the corresponding swap rate (representing the par yield curve). We can then write

$$\begin{aligned} 1 &= \sum_{i=1}^{n} v^*(t_0, t_i) R_n + v^*(t_0, t_n) - EL_n = \\ &= \sum_{i=1}^{n} v^*(t_0, t_i)(R_n - sr_n) + 1 - EL_n \end{aligned} \tag{2}$$

where we have added and subtracted

$$\sum_{i=1}^{n} v^*(t_0, t_i) sr_n = 1 - v^*(t_0, t_n)$$

We finally have

$$EL_n = \sum_{i=1}^{n} v^*(t_0, t_i)(R_n - sr_n) \tag{3}$$

or, in terms of *asset swap spread* (ASW)

$$ASW = \frac{EL_n}{\sum_{i=1}^{n} v^*(t_0, t_i)} = (R_n - sr_n) \tag{4}$$

Remark 3.1. *Consider that the development of debt could be obtained by setting*

$$D_i = D_{i-1} + \left(\frac{1}{v^*(t_{i-1}, t_i)} - 1 + ASW \right) D_{i-1} - S_i$$

to obtain

$$D_n = \frac{D_0}{v^*(t_0, t_n)}(1 + ASW)^n - \sum_{i=1}^{n} \frac{v^*(t_0, t_i)}{v^*(t_0, t_n)}(1 + ASW)^{n-i} S_i$$

which is the same as equation (1) if we consider

$$v(t_0, t_i) = \frac{v^*(t_0, t_i)}{(1 + ASW)^i}$$

4 Senior and Junior Public Debt

We now assume that public debt is broken into two tranches of different seniority. This means that in case of default, senior debt is repaid before junior debt, or, which is equivalent, junior debt absorbs losses until all of it is swept away before the senior tranche begins to be impaired. Denote D_0^S the senior debt at time t_0, and D_0^J the junior. We may perform the same recursion as above, assuming that senior debt is only issued at time t_0 and any future imbalance is funded by junior debt. So, we set $D_n^S = D_0^S$. We further assume that the amount of senior debt is issued at par with coupon R_n^S: typically, it would be such that the rate is risk free, and this is what we assume. As for junior debt, we obtain

$$D_n^J = \frac{D_0^J}{v^J(t_0,t_n)} - \frac{1}{v^J(t_0,t_n)} \sum_{i=1}^{n} v^J(t_0,t_i)\left(S_i - R_n^S D_0^S\right)$$

where

$$v^J(t_0,t_j) = \frac{v^*(t_0,t_j)}{(1+ASW_n^J)^j}$$

and ASW_n^J is the asset swap spread on junior debt. We now define the event of default by time t_n as $D_n > D_K$, as in the model before. Recalling that $D_n = D_n^J + D_n^S = D_n^J + D_0^S$, we then compute

$$\mathbb{P}\left(D_n^J + D_0^S > D_K\right) =$$

$$= \mathbb{P}\left(\frac{D_0^J}{v^J(t_0,t_n)} - D_0^J - \frac{1}{v^J(t_0,t_n)} \sum_{i=1}^{n} v^J(t_0,t_i)\left(S_i - R_n^S D_0^S\right) - (D_K - D_0^J - D_0^S) > 0\right)$$

$$= \mathbb{P}\left(\frac{D_0^J}{v^J(t_0,t_n)} - D_0^J - \frac{1}{v^J(t_0,t_n)} \sum_{i=1}^{n} v^J(t_0,t_i)\left(S_i - R_n^S D_0^S\right) - (D_K - D_0) > 0\right)$$

Now, assuming that junior debt is issued at par and applying the same algebra as before,

$$\mathbb{P}(D_n > D_K) = \mathbb{P}\left(R_n^J D_0^J + R_n^S D_0^S - \sum_{i=1}^{n} \omega_i^J S_i - \omega_n^J (D_K - D_0) > 0\right)$$

where we define

$$\omega_i^J = \frac{v^J(t_0,t_i)}{\sum_{i=1}^{n} v^J(t_0,t_i)}$$

If we want to express the values in terms of GDP we define accordingly

$$\eta_i^J = \omega_i^J (1+g)^i$$

In the end, we can define a distance to default in the model with tranching the same way as we do in the general model

$$DD^*(t_0,t_n) = -\frac{\bar{s}\sum_{i=1}^{n} \eta_i^J - d_0^J R_n^J - d_0^S R_n^S + \omega_n (d_K(1+g)^n - d_0)}{\sigma\sqrt{\sum_{i=1}^{n} {\eta_i^J}^2}}$$

where small capital letters denote that the value is written in terms of GDP. Now, remember that tranching the debt into tranches does not change the probability of a credit event on the debt as a whole, and the aggregate cost of debt remains the same. This implies $DD^*(t_0, t_n) = DD(t_0, t_n)$. Once such probability of default is given and the interest rate on senior debt is known, we can recover the spread of junior debt.
Of course, the result that tranching does not affect the overall value of debt relies on the assumption that no outside guarantee is provided to the senior tranche. Furthermore, the effect on borrower's discipline is not taken into account either. Both these effects are instead present in the Eurobond proposal, as we are going to discuss below.

5 Eurobonds

We now specialize the analysis above to the Eurobond project. We remind that this project consists of the establishment of a European Debt Agency that would raise funds from investors to provide senior funding to Governments. This way, part of the primary surplus of the European Union as a whole would provide a joint guarantee for the funding of local Governments. To summarize, the project consists of three building blocks:

- a seniority structure of Government liabilities: funding from the European Debt Agency would have priority of repayment in case of default of a country, or, which is the same, the funds provided by the European Debt Agency would be impaired by losses only after all other liabilities are swept away;
- cross-guarantee structure among Governments: the funding provided by the European Debt Agency to each country would be jointly guaranteed by all countries, and it would be like if each country would pledge part of its primary surplus to a newborn European debt market;
- surplus diversification across Governments: since surplus and deficits are not perfectly correlated across countries it may be that the impact of a worsening of public deficit in one country could be balanced by opposite movements in some of the others.

In order to understand the principles, let us define

$$s_{i,EUR} = \sum_{j=1}^{k} \alpha_j s_{i,j}$$

where k is the number of countries, $s_{i,j}$ denotes the primary surplus of country j at time t_i, and α_j is a weight of the GDP of the country out of the GDP of the European Monetary Union. We can now apply the model described above to represent the distance to default of Eurobonds. In this context, we define

d_k as the threshold limit (as a percentage of GDP) up to which Eurobonds can be issued for each country (through the EDA). We assume, as it is the case in most of the proposals, that this threshold would be formally defined by the constitution act of the EDA. While issuance of Eurobonds could take place gradually up to the threshold, here for simplicity we assume that Eurobonds are issued right from the start for a percentage of GDP $d_0^{EUR} = d_k$ and rolled over for the same quantity at time t_n. Then we compute

$$DD(t_0, t_n) = \frac{\bar{s}_{EUR} \sum_{i=1}^{n} \eta_i - d_0^{EUR} R_{EUR} + \omega_n d_0^{EUR} \left((1 + g_{EUR})^n - 1\right)}{\sigma_{EUR} \sqrt{\sum_{i=1}^{n} \eta_i^2}} \tag{5}$$

where $\bar{s}_{EUR}$ and σ_{EUR} denote the mean and volatility of the aggregate European primary surplus, while g_{EUR} is the average rate of growth of the Euro area. If one wants to disentangle the effect of diversification, it is sufficient to compute

$$\rho = \frac{\sigma_{EUR}}{\sum_{j=1}^{k} \alpha_j \sigma_j}$$

and the distance to default can be written

$$DD(t_0, t_n) = \frac{1}{\rho} \frac{\bar{s}_{EUR} \sum_{i=1}^{n} \eta_i - d_0^{EUR} R_{EUR} + \omega_n d_0^{EUR}((1 + g_{EUR})^n - 1))}{\sum_{j=1}^{k} \alpha_j \sigma_j \sqrt{\sum_{i=1}^{n} \eta_i^2}}$$

Now, for the proposal to be effective the supply of Eurobond must be calibrated to be riskless, so that one could be reasonably sure it could be rolled over at maturity. That way, it could be issued at the risk-free rate, so that we could set $R_{EUR} = sr_n$.

Let us now address on theoretical grounds the advantage that could come to a country issuing Eurobonds to replace part of domestic debt. This originates from two sources: the first is a possible decrease in the overall cost of debt. The second is the increase in the marginal cost of debt, or of the domestic part of debt, which is due to the seniority structure. Inasmuch as the latter effect provides an incentive to pursue a reduction of domestic debt by an increase in primary surplus, it may effectively reinforce the former.

In order to measure the first effect, we propose the following technique. We compute the expected loss of the Eurobond and we compare it with the sum of the expected losses of senior domestic bonds with the same features issued individually by each country. Of course, the difference between the expected loss of the sum of domestic bonds and that of the Eurobonds is the reduction of debt cost (cash collateral) that could be achieved with Eurobonds. This gain is due to cross-guarantee and diversification. The former is measured by the convexity of the probability function and the latter by correlation. The following simple two-country example explains.

Remark 5.1. *In order to represent the effect of the mutual guarantee of the Eurobond, consider two countries, with expected surplus $\bar{s}_1 > \bar{s}_2$ and volatility $\sigma_1 < \sigma_2$. So, country 1 is more virtuous than country 2, because its expected* *surplus is higher and more credible than that of the other country. The aggregate expected surplus is* $\alpha\bar{s}_2 + (1-\alpha)\bar{s}_1$*. Under the assumption of perfect correlation, we would have that its volatility would be* $\alpha\sigma_2 + (1-\alpha)\sigma_1$*. For the sake of simplicity, assume zero rate of growth for both countries, so that* $g = 0$*. Now assume that a bond with joint guarantee is issued by the two countries for an amount of* d_0 *of GDP. Furthermore, assume that the bond is assisted by a guarantee so that it pays the risk-free rate. This would have a distance to default equal to*

$$DD(\alpha s_2 + (1-\alpha)s_1) = \frac{\bar{s}_1 + \alpha(\bar{s}_2 - \bar{s}_1) - sr_n d_0}{\sigma_1 + \alpha(\sigma_2 - \sigma_1)}$$

The guarantee can be conceived as placing cash for the amount $\Phi(-DD)d_0$*, assuming that all debt may get lost in case of default. A question is how to measure the cross-guarantee and diversification effect. The natural measure is to compare the joint bond with individual bonds with the same features. Formally, denote* DD_i*,* $i = 1, 2$ *the distance to default of a debt issue of the same amount guaranteed by each country separately*

$$DD(s_i) = \frac{\bar{s}_i - sr_n d_0}{\sigma_i}$$

with cash guarantee equal to $\Phi(-DD_i)d_0$*. It is then clear that*

$$DD(s_2) < DD(\alpha s_2 + (1-\alpha)s_1) < DD(s_1)$$

Now, a natural measure of the cross-guarantee is given by the convexity of the default probability: that is, the cross-guarantee of the joint bond adds value if

$$\Phi\left(DD(\alpha s_2 + (1-\alpha)s_1)\right) < \alpha\Phi\left(DD(s_1)\right) + (1-\alpha)\Phi\left(DD(s_2)\right)$$

Beyond cross-guarantee, finally, the effect of diversification will be given by

$$\Phi\left(-\frac{1}{\rho}\frac{\bar{s}_1 + \alpha(\bar{s}_2 - \bar{s}_1) - sr_n d_0}{\sigma_1 + \alpha(\sigma_2 - \sigma_1)}\right) < \Phi\left(-\frac{\bar{s}_1 + \alpha(\bar{s}_2 - \bar{s}_1) - sr_n d_0}{\sigma_1 + \alpha(\sigma_2 - \sigma_1)}\right)$$

The second effect, that is the increase of marginal cost of new debt, is a standard result stemming from the seniority structure which is part of the Eurobond proposal. Again, consider what would happen if a country substituted a part of its domestic debt with a new form of debt which is senior with respect to the rest of debt. In this case, the decrease in the cost of senior debt would be exactly compensated by an increase in the cost of junior debt, so that the overall cost of debt would remain unchanged. If we now extend this argument to debt substitution with Eurobonds, it is clear that the reduction of cost provided by senior funding will be larger than that obtained with domestic tranching, so it will be less than compensated by the increase in the cost of junior debt (some figures on possible gains are reported in the section below). However the increase of marginal cost of debt remains unchanged.

6 A Numerical Proposal

We present here some first calculations on the model, with the twofold aim of: i) showing how a Eurobond issue could be structured and ii) gauging the possible impact of the Eurobond project on European debt. We begin with the structure of the Eurobond. Just for the sake of illustration, we assume the technical feature to be:

- Ten year maturity, issued at par;
- Maximum size of the tranche equal to 40% of GDP: no Eurobond will be issued beyond this limit;
- Interest rate payments equal to the risk-free rate.

The first two points are simply a matter of the structure of the security, which is kept at the simplest level, that is a *plain vanilla* bond. The third requirement, that it must pay the risk-free rate, requires an ancillary structure of guarantees that could make the bond riskless. In what follows, we will in turn address the questions of how to design the product to make it riskfree and how to measure the pros and cons for the countries willing to use them to substitute part of the debt.

6.1 Designing the Eurobond

In order to "structure" the product, we proceed in two steps. In the first, we design the product without guarantees, and evaluate its expected loss. In the second step we design provisions to abate the expected loss to zero: we call these ancillary clauses *credit enhancements*, borrowing the term from the securitization literature. Without getting into sophisticated financial engineering techniques, here we restrict the analysis to the most straightforward of these provisions, namely that the Governments participating in the EDA will post cash to reduce the expected loss to zero.
Let us begin by computing the expected loss of a Eurobond with this structure. Based on historical information before the crisis (the period from 1999 to 2007) we have that the average primary surplus of the aggregated Euro area is around 1.15%. Surplus volatility is 1.23%, while the average of surplus volatilities is 1.52%. This means that primary surpluses are highly correlated, with a correlation ratio of 81%. Nevertheless, we will see that the diversification effect is quite sizable. As a final hypothesis, we assume an average growth of nominal GDP of 3.5%, based on a 2-2.5% range for inflation and 1-1.5% for real growth. Let us assume that Eurobonds are issued for a share of 40% of the GDP of the Euro area. We can compute the probability of trajectories of Europe-wide primary surplus insufficient to cover the interest cost. With our model, and using the market data from August 1st 2011 for the yield curve, we compute a distance-to-default figure (see equation (5)) of 3.26, corresponding to a cumulated probability of default of 0.06% on a 10 year horizon. Assuming that 3665

billions euro (about 40% of the Euro area debt in 2010) were issued, this would lead to an expected loss figure (assuming the extreme 100% LGD scenario) of little less than 2 billions Euros (about 0.02% of GDP). This gives a measure of the credit enhancement provision required: Governments of the Euro area could be required to post a sum like this in cash as a guarantee. This would actually make the Eurobonds risk-free in this scenario.
We now subject the credit enhancement results above to a stressed scenario. The sources of risk are four

- a decrease of expected surplus;
- an increase of the surplus volatility;
- an increase of the correlation of surpluses across countries;
- a decrease in the expected growth of GDP;

In table 1 we report scenarios for each source of risk. For the sake of brevity, scenarios on the increase of volatility and correlation are gathered in one single scenario. In fact, an increase in correlation of the primary surpluses of the Euro area impact on the probability of default through an increase in volatility of the aggregated surplus.

In the first scenario, we assume a decrease of expected primary surplus (as a percentage of GDP) to 0.90%. In this case, the credit enhancement increases to around 16 billions. The same effect results if we instead assume an increase in volatility, setting it to 1.52% (corresponding to perfect dependence of primary surpluses across countries). A reduction of GDP growth has also an effect of 10 billions on the cash guarantee required. On the relevance of growth we will digress more in detail below. Here we just sum up the results in an extreme scenario, featured by a decrease of expected surplus and of GDP growth along with an increase in volatility/correlation, everything in line with the assumptions above. In this scenario we have a relevant impact of 155 billions (1.69% of GDP). This may seem costly, but remember that we also made the extreme assumption of 100% LGD in case of Eurobond default: if we were to stick to the assumption of 60% LGD commonly made in the CDS market, providing insurance against default of Eurobonds would cost about 93 billions. If we consider that Euro area countries are committed to put 80 billions as paid-in capital of the ESM that will be launched in 2013, we can see that making safe the issuance of Eurobonds would cost little more than that.[5]

Table 1

[5] Remember also that the potential liability generated by the ESM for the Euro area governments amounts to the 700 billion subscribed capital, including callable capital and guarantee in addition to paid-in capital.

Stress Tests of the Eurobond Cash Guarantee

Expected Surplus	Surplus Volatility	GDP Growth	Cash Guarantee
1.15%	1.23%	3.50%	2,059,409,033
1.15%	**1.52%**	3.50%	15,370,146,777
0.90%	1.23%	3.50%	16,357,313,780
1.15%	1.23%	**3.00%**	10,247,097,230
0.90%	**1.52%**	**3.00%**	155,851,316,069

6.2 The relevance of GDP growth

In the policy debate on debt sustainability and the public finance crisis a major role has been played by growth. The same occurs in our model. Given the same expected value and volatility of primary surplus, expressed in terms of GDP, a higher growth of GDP would correspond to a higher primary surplus in nominal terms and a reduction of the weight of nominal debt. Of course, there is no need to remind that primary surplus and GDP growth cannot be considered independent, since an increase in primary surplus is likely to bring about a decrease in GDP growth, as we know from standard macroeconomics. So, existence of this trade-off must be always kept in mind when going through the results that will follow, and in view of actual implementation of the model. In order to highlight the relevance of GDP growth in our model, we present three figures with level curves representing the same degree of risk of the Eurobond. In the first two figures, this is measured by the default probability of the Eurobond.
In Figure 1, we set this default probability at 1% and we draw the couples of values of expected primary surplus and GDP growth that are consistent with this value (keeping all the other parameters at the same levels of the base scenario described above).The level curves are drawn for three different levels of the amount issued (40, 50 and 60% of GDP). The figure shows, and this is no surprise, that a higher rate of GDP growth allows to maintain default probability at the 1% level with lower levels of expected primary surplus. The slope of the relationship increases with the amount issued, so that the level curves cross. The rationale of this result is that of Remark (2.1), that is the sign of the relationship between level of debt and default probability depends on the difference between the interest rate and the rate of growth of GDP. As for the numbers, it turns out that with a rate of growth of nominal GDP around 3.20%, in all cases a primary surplus of 0.90% of GDP would be sufficient to keep the probability of default at 1%. If the rate of growth would mount to 4%, then a primary surplus of 0.45% would be required in the case of 60% issue.

In Figure 2 we represent a standard level curve, for the same 1% default probability level, linking mean and volatility of primary surplus. This is important because it describes the trade-off between fiscal policy announcements and their degree of credibility. If the volatility of future primary surpluses is higher (that is they are more uncertain), a higher level of expected surplus is required to maintain the same default probability. The relationships are linear,

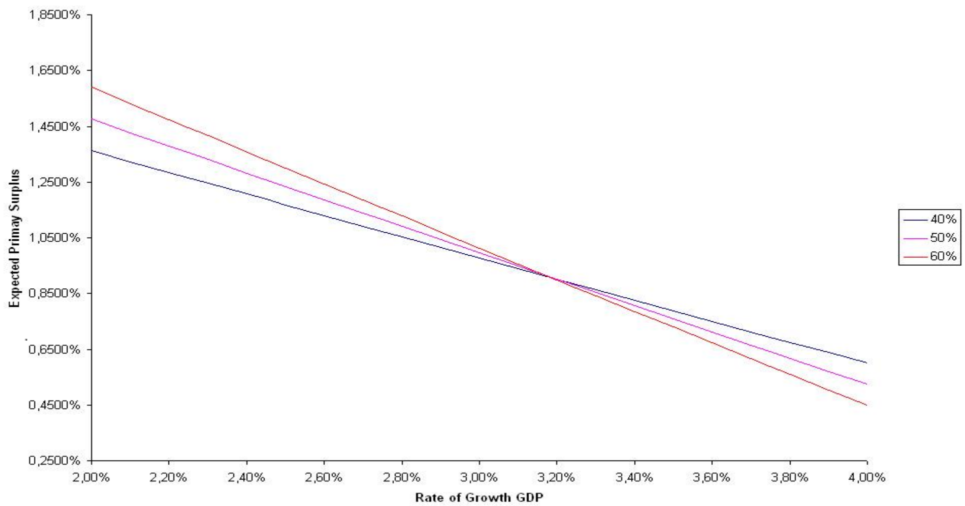

Figure 1: Level curves of GDP growth rates and expected surpluses for default probability equal to 1%

and for a 1% increase of volatility about 0.735% increase in expected surplus is required to maintain the default probability at 1%: this ratio increases with the probability of default. Even in this case, GDP growth matters, bringing about a shift of the curve downward (for higher growth rates): namely, a 1% increase of GDP growth would require a saving of 0.38% primary surplus to keep the same level of default probability.

In Figure 3, finally, we report three level curves defined in terms of cash amount guarantee that could be posted to hedge the risk of the Eurobonds. The different curves refer to amounts ranging from 0.8 to 80 billions. The latter choice is particularly important, because it casts a link to the amount that will be posted to guarantee the ESM since 2013: notice that in this case, even with a rate of growth of GDP of 3%, an expected primary surplus of 0.86% would be sufficient to guarantee the Eurobond issue.

6.3 Benefits and Costs: a Two-Country Model

We now check the effects of Eurobonds on the countries participating in the scheme. Without getting explicit, we simply assume two hypothetical countries

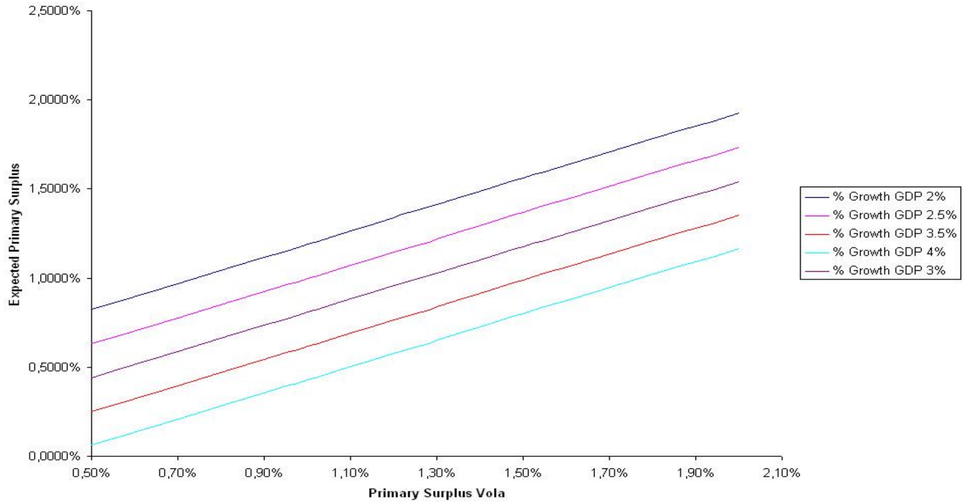

Figure 2: Level curves of primary surplus expected value and volatility for default probability equal to 1%

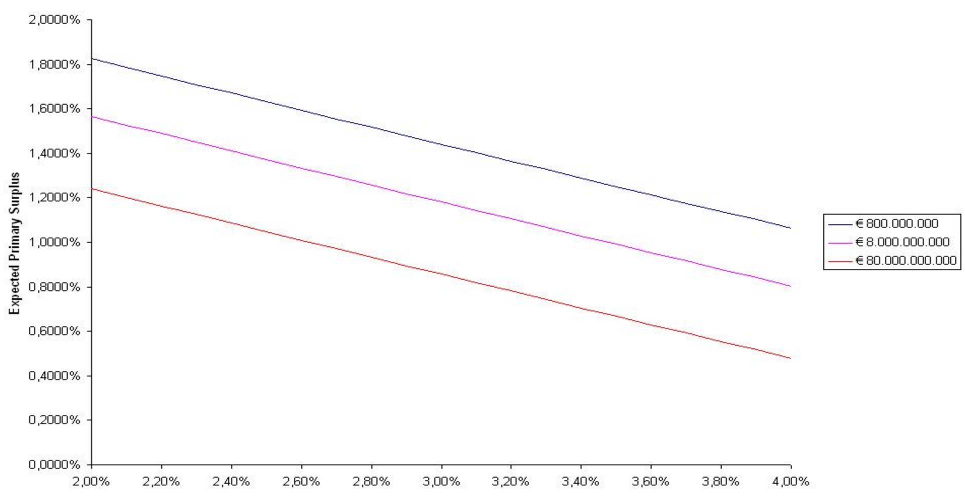

Figure 3: Level curves of GDP growth rates and expected primary surpluses for different level of cash guarantees (0.8, 8 and 80 billions

(or groups of countries in the European Union). The first country is called Good, it has a manageable debt, on which it pays almost no spread. Although it is assumed to have exploited its whole debt capacity, so that it cannot increase its debt any further, it will be able to roll-over its outstanding debt almost for sure: this is based on expectations of a sound and quite credible fiscal policy. In terms of numbers, we assume an initial level of debt of 85% which is bound to remain unchanged at maturity: $d_0 = d_K = 85\%$, while expected surplus is 1.20% and the volatility is 1.52%. The second country is called Bad, it has a huge debt and a primary surplus which is lower and less credible. Of course, in this case debt is bound to grow up to a threshold when it will not be rolled over, and because of the probability of that event it will pay a premium for default risk. In numbers, we set the initial debt d_0 equal to 120% and the threshold d_K at 150%. As for primary surplus, we assume the expected value to be 0.6% with 1.52% volatility as for the Good country: this means that in percentage terms fiscal policy is more uncertain. Finally, we assume that both the Good and Bad country have the same dimension in terms of GDP.

We now address the question of the effect of the Eurobond scheme on the average cost of debt. The method we apply is as follows. We first assume that each country domestically performed the same tranching of debt as that implemented with the Eurobond structure, and then we compare the dimension of the cash credit enhancement, that is the amount that should be paid to make it safe. If this were lower than that required for Eurobonds, then the country would loose from the introduction of Eurobonds. On the contrary, if the credit enhancement were more costly than the deposit required for Eurobonds, then Eurobonds would imply a reduction of the average cost of debt equal to the difference between the two credit enhancements.

As for the Eurobond scheme, we first assume that the credit enhancement under the Eurobond scheme would be based on a scenario with expected surplus 0.90% and volatility equal to 1.52% (the average of the volatilities of the two countries). So, in this setting we assume perfect correlation of the surpluses of the good and bad country. Notice that this corresponds to the most conservative scenario in Table 1. We assume again 100% LGD. In this case, the cash deposit required, for 40% of GDP senior debt, would amount to 1.7% of GDP. If we maintain the same conservative assumption concerning GDP growth and we compute the same tranching in a domestic setting, we obtain a credit enhancement requirement of: i) 0.4% for the Good country; ii) 7.7% of GDP for the Bad country. If both the Good and the Bad country were required to post the same amount of cash, that is 1.7% of GDP, then the Good country would have a cost of 1.3% of GDP, and the Bad country would have 6.00% benefit. The cross-country guarantee from the Good country to the Bad would then lower the overall cost of senior debt from about 4% to about 1.7%, with a saving around 2.3%.

If one now makes the more realistic assumption of some diversification of aggregate primary surplus, then there is an additional net benefit to be shared among countries. Even with the high 81% correlation of surpluses, this additional benefit is as large as 1.03% of the GDP of the whole Euro area (which

amounts to almost 94 billions Euro): the cash deposit would be 0.67% of GDP instead of 1.7%.
To complete our example, consider what could be a reasonable allocation of the insurance burden between country Good and Bad. The Good country could be asked to post at most the same amount of cash collateral as with domestic tranching (0.4% of GDP); for every guarantee charge lower than this the Good country would benefit. As for the Bad country, in this case it would benefit anyway, because of the huge difference between the cost of domestic senior bonds and that of Eurobonds. In fact, even if the Bad country were to place all the required guarantee, that is 1.34% of its GDP (0.67% / 0.5) it would maintain a large margin of gain, while Eurobonds would be free for the Good country.

Remark 6.1. *Notice that existence of benefits from cross-guarantee and diversification is not granted in all cases. In the example at hand, it turns out that no benefit would persist in the hypothesis of a GDP nominal growth of 2.10%. Below that level of growth, domestic tranching would be less costly.*

We now come to the second effect of the Eurobond scheme, that is the fact that seniority of the Eurobonds would rise the marginal cost of issuing junior debt, providing an incentive to financial discipline. Also in this case, we calibrated the effect on a scenario of 3% average growth of nominal GDP. It turned out that Good country would experience a credit spread on junior debt of about 47 basis points, up from the 25 basis points that it would experience without tranching. The discipline effect would be a little higher on the Bad country, where the credit spread in this low growth scenario would rise to 2.37%, 37 basis points higher than the credit spread without tranching. In both cases the effect is only reported for reference, because fiscal policy is assumed invariant.

7 Policy Considerations

We collect here the main policy remarks that can be extracted from our model and the application to the Eurobond project. We organize the arguments in the following points:

- **The importance of pooling.** What is the advantage of issuing senior Eurobonds over senior domestic bonds? Notice that the answer to this question is not that some Governments could not afford it. Indeed, one could conceive a European mechanism providing funds to support senior domestic bond programs set up by each country individually. Differently from Eurobonds, these programs would be guaranteed by the expected primary surplus of each country. With respect to these programs, Eurobonds provide the advantage of pooling. Even though the primary surpluses of the Euro area are highly correlated (about 81% average correlation), the pooling effect reduces the volatility of the primary surpluses of the Euro area from 1.52% to 1.23%. We saw that this provides the same advantage as a reduction of expected surplus from the historical average (1999-2007) of 1.15% to a level around 0.90%. So, in a period in which there is a trade-off between expansionary fiscal policies to contrast the recession and the debt sustainability problem, issuing Eurobonds would allow to save about 0.25% of GDP of primary surplus that could be devoted to expansionary fiscal policies. We also saw that this advantage requires a fiscal policy of positive primary surpluses and some average growth of GDP over the long term.

- **The importance of fiscal credibility.** Apart from the effect of pooling, a reduction of volatility must come from credible fiscal policies. With respect to this, the fears of the German authorities that Eurobonds could decrease fiscal discipline, and so increase volatility, may be justified. However, we showed that the marginal cost of debt − namely the rate paid on national junior bonds − may increase substantially. This is the main tool by which Eurobonds would actually enforce fiscal discipline. In order to preserve this principle and to make it more effective, one could think of including ancillary rules like the following: i) a clause that a percentage of domestic debt must be always maintained, so that the market for junior bonds could provide a signal of the direction and credibility of the fiscal stance of a country; ii) designing and enforcing regulation on the transparency of public balance sheets; iii) imposing lower limits to the primary surplus of the countries issuing Eurobonds. Notice that point iii) is actually already enforced under the Stability Pact.

- **The importance of growth.** We saw that the relevance given in the debate to the problem of GDP growth is a well taken point. We only remind that as far as our analysis is concerned, we are talking about growth of *nominal* GDP. So, a given rate of growth may at the same time represent an economy which is highly growing in real terms as well as an economy in stagflation. Of course, the two cases are different, but the difference could only be caught in a general equilibrium approach, and anyway would show up in other variables of the model. As for our analysis, we chose a base scenario of 3.50% average nominal growth on a ten year horizon. This scenario would make the Eurobond project largely feasible. As an example, if the 80 billions allocated to the paid-in capital of the ESM were used to guarantee the Eurobonds, with this rate of growth an expected surplus of 0.67% would suffice to make the Eurobond issue risk free.

- **The importance of quantity.** Where to draw the line? In part of the analysis that we did not report here it turned out that in an ordinary scenario of GDP growth at 3.5% there was not much of a difference between issuing 40% or 60% of GDP. However, in the hypothesis of an adverse scenario with slow growth, low and uncertain primary surpluses (which are typically associated to it), issuing 60% could be risky. We found that the cash guarantee amount required to hedge a scenario like this could be easily as high as 300 billions. Something could be done if one resorted to special financial innovation tools such as GDP-linked bonds for part of the issue, but for the time being it seems wise to stick to a safe choice of 40%.

- **The importance of governance.** Who shall decide the amount? The main cons of the Eurobond project is that it will bring about losses to the private sector. To understand this, take the example of Italy. Using market data from August 1st, we assumed to issue Eurobonds for 40% of GDP to substitute domestic debt, and we computed that the return on the remaining 79%, which would become junior: the return on junior debt would increase by about 134 basis points. The shock is not astonishing, since it is quite similar to what we saw at the beginning of August 2011. In this case, it would not weight on the whole debt; however it would represent a consistent loss anyway. If we assume a conservative value of the duration of the whole Italian government debt of 3, we can compute that, as a first order approximation, the loss would be 1,35% X 3 X 79% = 3.176% in terms of GDP, which amounts to almost 50 billions Euro. It is clear that the decision as to who must have the right and responsibility to decide the amount and the schedule cannot be the country alone. There is then a need to design the governance of the European body (presumably the European Debt Agency), which should balance the benefits coming from the issuance of Eurobonds with the costs that would be inflicted to the private sector. Decisions must be made concerning the degree of discretion and the accountability of the body. Furthermore, technical rules could be added to ease the cost that is inflicted to private investors. The first two examples that come to mind are: i) reducing the duration of the junior bonds, by preferably substituting fixed rate bonds with longer maturities and low coupons (and low liquidity, since liquidity risk is priced in the market): so, if in the example above the amount of junior debt remaining had duration 1, the loss would be little more than 1% of GDP; ii) imposing that the Eurobond issue should be balanced with a fiscal plan leading to the reduction of the spread on junior debt.

- **The importance of being earnest.** What are Eurobonds for? In the policy debate, it is not always clear whether these products should be used for debt management purposes, in particular to support countries in financial crisis, or as a funding tool of a future common fiscal policy. One can be in favor of one or the other application of Eurobonds. Our setting provides a general technical framework that can be applied to both of goals. Of course, the allocation of the guarantee that we propose, based on the riskiness of the country, is well suited to the use of Eurobonds to substitute domestic debt. It is in fact conceivable that Eurobonds issued for infrastructures should also take into consideration the benefits accruing to each country from the infrastructure itself, on top of the credit risk principle.

8 Concluding remarks

In this paper we have explored the idea of introducing Eurobonds as a way to reduce the average cost of servicing public debts in the euro area. We have employed a structural model of sovereign credit risk, which has been applied to analyze the impact of Eurobonds and of domestic tranching as well. While the main ingredients of the Eurobond scheme have been summarized in the Introduction, let us stress here our main findings.

- Eurobonds are able to lower the average cost of debt for sovereign borrowers by exploiting cross-guarantee and diversification effects: although the path of national deficits/surpluses are highly correlated, this gain can be quite substantial. In particular, the Eurobond scheme allows high risk countries to significantly reduce the amount of collateral needed to issue riskless senior debt. The only scenario that could jeopardize these gains is one with low growth and scarce fiscal discipline.

- The volatility of the primary balance of the public sector is a crucial parameter in the determination of the amount of collateral to be pledged to make Eurobonds risk free. Lower volatility enables countries to post less collateral for given expected surplus, or equivalently to run a lower average surplus for given collateral. This result points to the relevance of credibility of stabilization plans for public finance. Under this perspective any tool to reduce volatility, like fiscal rules embedded in Constitutions, are welcome.

- GDP growth has a remarkable role. The estimates of the cash collateral and of the expected surplus needed to implement the Eurobond scheme, as well as its economic convenience, are very sensible to the assumed rate of growth of nominal GDP.

- The increase of the interest rate on junior national bonds may be quite substantial, implying an equivalent increase of the marginal cost of public debt. This should provide the right incentives to national governments to curb their debt, thus removing any moral hazard effect possibly due to the cross-country guarantee.

  While our results support the Eurobond project, some issues deserve further analysis, for example:

- The holders of national debt will be hurt by the fact that their claim becomes junior to the European Debt Agency. The preferred status given to the EDA could bring about a substantial price decrease. The impact of this loss on financial intermediaries should be carefully considered, and some ways to limit such loss should be found (some suggestions are made above).

- It is worth noting that the Eurobond project should be considered also under a scenario where some countries might decide to opt out. Measuring the effects of introducing Eurobonds in a subset of the Eurozone is a matter for future research.

- The issue of market liquidity was not addressed in this paper, and would add to the debt cost reduction that we studied here. Ironically, while this is an advantage from an economic point of view, it could be a cons from the political one. In fact, the birth of a new large market where investors could switch in periods of market stress could be a competitor for the Bund, preventing Germany from exploiting such periods of crisis for the reduction of its interest burden.

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